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# Complementary Acyclic Weak Domination Preserving Sets

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**Abstract:** Let G = (V, E) be a simple graph. A subset D of V(G) is called complementary acyclic weak domination preserving set of G (c-awdp set of G) if < V – D > is acyclic and $\gamma_w(\langle D\rangle) = \gamma_w(G)$. The minimum cardinality of a c-awdp set in G is called the complementary acyclic weak domination preserving number of G and denoted by c-awdpn(G). A c-awdp set of G of cardinality c-awdpn(G) is called a c-awdpn-set of G. In this paper, we introduce and discuss the concept of complementary acyclic weak domination preserving sets.
**Keywords**: Complementary acyclic weak domination preserving set,complementary acyclic weak domination preserving number.

## I. INTRODUCTION

By a graph we mean, simple and undirected graph G(V, E) where V denotes its vertex set and E its edge set. Degree of a vertex u is denoted by d(u). The maximum degree of a graph G is denoted by Δ(G). We denote a cycle on n vertices by $C_n$, a path on n vertices by $P_n$ and a complete graph on n vertices by $K_n$. A graph G is connected if any two vertices of G are connected by a path. A maximal connected sub graph of a graph G is called a component of G. The number of components of G is denoted by ω(G). The complement $\overline{G}$ of G is the graph with vertex set V in which two vertices are adjacent if and only if they are not adjacent in G. A graph G is said to be acyclic if it has no cycles. A tree is a connected acyclic graph. A bipartite graph is a graph whose vertex set can be partitioned into two disjoint non empty sets $V_1$ and $V_2$ such that every edge has one end in $V_1$ and another end in $V_2$. A complete bipartite graph is a bipartite graph where each vertex of $V_1$ is adjacent to every vertex in $V_2$. The complete bipartite graph with partitions of order $|V_1|$ = m and $|V_2|$ = n, denoted by $K_{m,n}$. A star denoted by $K_{1,n-1}$ is a tree with one root vertex and n-1 pendant vertices. A bistar, denoted by D(r,s) is the graph obtained by joining the root vertices of the stars $K_{1,r}$ and $K_{1,s}$. A wheel graph denoted by $W_n$ is a graph with n vertices formed by joining a single vertex to all vertices of $C_{n-1}$. A helm graph, denoted by $H_n$ is a graph obtained from the wheel $W_n$ by attaching a pendant vertex to each vertex in the outer cycle of $W_n$. Corona of two graphs $G_1$ and $G_2$, denoted by $G_1 \circ G_2$ is the graph obtained by taking one copy of $G_1$ and $|V(G_1)|$ copies of $G_2$ in which $i^{th}$ vertex of $G_1$ is joined to every vertex in the $i^{th}$ copy of $G_2$. If D is a subset of V, then ⟨D⟩ denoted the vertex induced sub graph of G induced by D. The open neighborhood of a set D of vertices of graph G, denoted by N(D) is the set of all vertices adjacent to some vertex in D, and $N(D)\cup D$ is called the closed neighborhood of D, denoted by N[D]. The diameter of a connected graph is the maximum distance between two vertices in G and is denoted by diam(G). A cut-vertex of a graph G is a vertex whose removal increases the number of components. A vertex cut of a connected graph G is a set of vertices whose removal results in a disconnected graph. The connectivity or vertex connectivity of a graph G, denoted by k(G) (where G is not complete) is the size of a smallest vertex cut. A connected sub graph H of a connected graph G is called a H-cut if ω(G-H) ≥ 2. For any real number $\lfloor x\rfloor$ denotes the largest integer less than or equal to x.

A subset D of V is called a dominating set of G if every vertex in V-D is adjacent to at least one vertex in D. The domination number γ(G) of G is the minimum cardinality taken over all dominating set D in G. A dominating set D of G is called a weak dominating set of G if for every $v \in V - D$ there exist a vertex $u \in D$ such that $uv \in V(G)$ and $d(u) \le d(v)$. The minimum cardinality taken over all weak dominating sets is the weak domination number and is denoted by $\gamma_w(G)$.

In this paper, we use this idea to develop the concept of complementary acyclic weak domination preserving number of a graph.

**Complementary Acyclic Weak Domination Preserving:**

**Definition: 2.1** A subset D of G is called a complementary acyclic weak domination preserving set of G (c-awdp set of G) if $\langle V-D\rangle$ is acyclic and $\gamma_w(\langle D\rangle)=\gamma_w(G)$. The minimum cardinality of a c-awdp set in G is called the complementary acyclic weak domination preserving number of G and is denoted by c-awdpn(G). A c-awdp set of G of cardinality c-awdpn(G) is called a c-awdpn- set of G.

**Example: 2.2**


**Fig 2.1**

{$v_1$,$v_2$} is a complementary acyclic weak domination preserving set of G.

**Remark: 2.3** Let G be a simple graph. If c-awdp set of G is independent, then $\gamma$(G) = 1.

**Remark: 2.4** Since V(G) is c-awdp set, the existence of a c-awdp sets is guaranteed in any graph.

**Remark: 2.5** The c-awdp set property is superhereditary, since any super set of a c-awdp set is a c-awdp set. Hence a c-awdp set is minimal if and only if it is 1-minimal.

## II. MAIN RESULT

**c-awdpn for standard graphs**

1. c-awdpn($K_2$) = n – 1.
2. c-awdpn($K_n$) = n – 2, $n \geq 3$
3. c-awdpn($K_{1,n}$) = n.
4. c-awdpn($K_{m,n}$) = max {m, n}.
5. c-awdpn($D_{m,n}$) = m +n – 2.

**Theorem: 3.1** For any Path $P_n$, n $\geq$ 3

$$c-awdpn(P_n)=\left\lceil \frac{n}{3}\right\rceil \text{if } n\equiv 1(\text{mod }3)$$

$$=\left\lceil \frac{n}{3}\right\rceil +1 \text{if } n\equiv 0 \text{ or } 2(\text{mod }3)$$

**Theorem: 3.2** For any cycle $C_n$, n $\geq$ 3

$$c-awdpn(C_n)=\left\lceil \frac{n}{3}\right\rceil \text{if } n\equiv 1(\text{mod }3)$$

$$=\left\lceil \frac{n}{3}\right\rceil +1 \text{if } n\equiv 0 \text{ or } 2(\text{mod }3)$$

**Theorem: 3.3** For any wheel $W_n$, $n \geq 4$

$$c-awdpn(W_n)=\frac{n}{2} \text{ if } n \text{ is even}$$

$$=\frac{n-1}{2} \text{ if } n \text{ is odd}$$

**Observation 3.4** A c-awdpn-set of a connected graph G need not induce a connected sub graph.

**Example: 3.5**

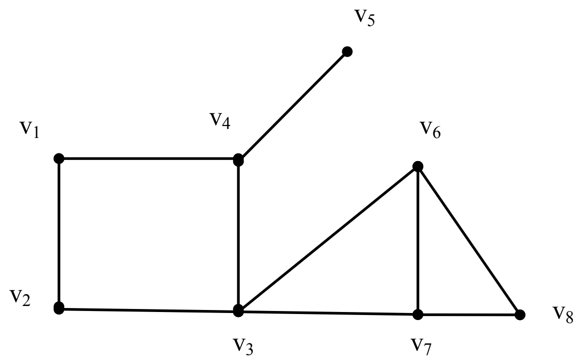


**Fig 3.1**

D = {$v_2$,$v_5$,$v_8$} is a c-awdpn-set which is disconnected.

**Observation: 3.6** $\gamma_w(G) = c - awdpn(G)$ if and only if any c-awdpn-set induces a complete subgraph.

**Proof:** By hypothesis $c - awdpn(G) = \gamma_w(G)$. Let D be a c-awdpn-set of G.

$c - awdpn(G) = |D| = \gamma_w(G) = \gamma_w(\langle D\rangle$.

Therefore $\langle D\rangle$ is a complete sub graph.

Conversely, let any c-awdpn-set induces a complete sub graph. Let D be a c-awdpn-set of G.

$|D| = \gamma_w(\langle D\rangle = \gamma_w(G)$.

That is $c - awdpn(G) = \gamma_w(G)$.

**Example: 3.7**

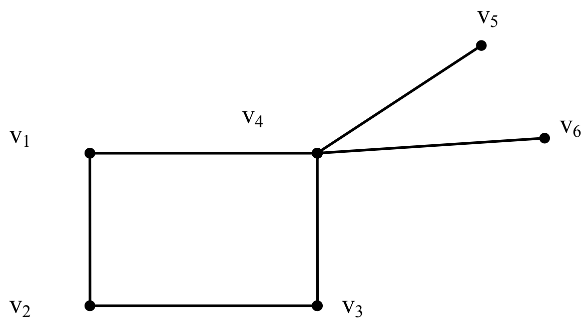


**Fig 3.2**

D={$v_2$,$v_5$,$v_6$} is a weak dominating set.

$\gamma_w(\langle D\rangle) = 3$ and also D is a c-awdpn-set.

**Remark: 3.8** If $\gamma_w(G) = \gamma(G)$, then $wdpn(G) = \gamma_w(G)$. But c-awdpn(G) may be greater than $\gamma_w(G)$.

**Example: 3.9**

D = {$v_2$, $v_5$, $v_6$}, $\gamma_w(G) = \gamma(G) = 2$ and c-awdpn(G) = 3.

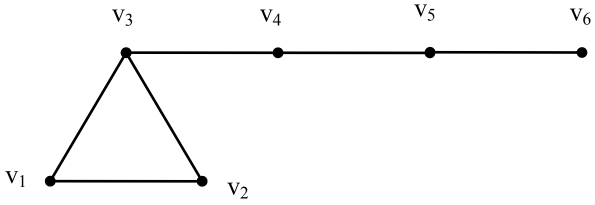


**Fig 3.3**

**Definition: 3.10** A subset D of G is called a minimal c-awdp set of G if D is a c-awdp set of G and no subset of D is a c-awdp set of G.
**Theorem: 3.11** Let D be a c-awdp set of G. D is minimal if and only if for any u in D, either V – (D – {u}) contains a cycle or $\gamma_w(\langle D-\{u\}\rangle) \prec \gamma_w(G)$.
**Proof:** obvious.

**Definition: 3.12** The maximum cardinality of a minimum c-awdp set of G is called the upper c-awdp number of G and is denoted by c-awdpN(G).
**Remark: 3.13** There are graphs G with c-awdpn(G) < c-awdpN(G).

## III. FINE C-AWDP GRAPHS

**Definition: 4.1** A graph G is a fine c-awdp graph if all minimal c-awdp sets have the same cardinality.
**Example: 4.2** (i) $K_{n,n}$ is a fine c-awdp graph.
(ii) All cycles are fine c-awdp graphs.

**Theorem: 4.3** Let $G_1$ and $G_2$ be graphs. Suppose $G_1$ contains a cycle and $G_2$ is acyclic and $\gamma_w(G_1) \le \gamma_w(G_2)$. Then $G_1 \cup G_2$ is a fine c-awdp graph if and only if $G_2$ is a fine c-awdp graph and all minimal c-a sets of $G_1$ have equal cardinality.
**Proof:** Suppose $G_1$ contains a cycle and $G_2$ is acyclic and $\gamma_w(G_1) \le \gamma_w(G_2)$. Any c-awdp set of $G_1$ is a c-a set of $G_1 \cup G_2$ but it is not a wdp set $G_1 \cup G_2$. Let $D_2$ be a minimal c-awdp set of $G_2$ and $D_1$ be a minimal c-a set of $G_1$. Clearly, $\gamma_w(D_2) = \gamma_w(G_2) \succ \gamma_w(G_1) \ge \gamma_w(D_1)$. Therefore $D_1 \cup D_2$ is a minimal c-awdp set of $G_1 \cup G_2$. Let $D = D_1 \cup D_2$, where $D_1 \subseteq V(G_1)$ and $D_2 \subseteq V(G_2)$. Since D is a c-a set of $G_1 \cup G_2$, $D_1$ is a c-a set of $G_1$ and $D_2$ is a c-a set of $G_2$. $\gamma_w(D) = \gamma_w(D_1 \cup D_2) = \max\{\gamma_w(D_1), \gamma_w(D_2\} = \gamma_w(G_1 \cup G_2) = \gamma_w(G_2)$.If $\gamma_w(D_1) \succ \gamma_w(D_2)$ then $\gamma_w(D_1) = \gamma_w(G_2) \succ \gamma_w(G_1)$, a contradiction, since $D_1$ is a subset of $V(G_1)$. Therefore $\gamma_w(D_1) \le \gamma_w(D_2)$.Therefore $\gamma_w(D_1) = \gamma_w(D_2) = \gamma_w(G_2)$. Therefore $D_2$ is a c-awdp set of $G_2$ and $D_1$ is c-a set of $G_1$. Since D is minimal, $D_1$ and $D_2$ are also minimal. Thus, $G_1 \cup G_2$ is a fine c-awdp graph if and only if $G_2$ is a fine c-awdp graph and all minimal c-a set of $G_1$ have equal cardinality. Similar argument can be given if $G_1$ contains a cycle and $G_2$ is a cyclic.

## IV. CONCLUSION

We found complementary acyclic weak domination preserving number for some standard graphs and general graphs.

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